

# EFFECTIVE AMBIENT VIBRATION TESTING FOR VALIDATING NUMERICAL MODELS OF CONCRETE DAMS

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## SUMMARY

Ambient vibration tests were conducted on a 56 metre high concrete gravity dam to measure its modal properties for validating a finite element model of the dam–reservoir–foundation system. Excitation was provided by wind, by reservoir water cascading down the spillweir, and by the force of water released through outlet-pipes. Vibrations of the dam were measured using accelerometers, and 3-hour data records were acquired from each location. Data were processed by testing for stationarity and rejecting non-stationary portions before Fourier analysis. Power spectra with low variance were generated from which natural frequencies of the dam were identified clearly and modal damping factors estimated. Modal analysis of the frequency response spectra yielded mode shapes for the six lowest lateral modes of vibration of the dam. The finite element model for the dam was analysed using EACD-3D, and the computed mode shapes and natural frequencies compared well with the measured results. The study demonstrates that ambient vibration testing can offer a viable alternative to forced vibration testing when only the modal properties of a dam are required. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: ambient; dynamic; tests; concrete; gravity; dams

## 1. INTRODUCTION

Dynamic tests are conducted on dams to determine their modal and dynamic response characteristics for validating numerical models. Both ambient<sup>1–4</sup> and forced vibration<sup>5–8</sup> tests have been employed for these purposes. Ambient vibration tests utilise environmental forces which cannot be measured, so only estimates of natural frequencies of vibration, mode shapes and modal damping factors can be made. In forced vibration tests, the excitation force is controlled, so force–response relationships can be determined leading to more accurate modal properties.

Environmental forces at dam sites can include water, either discharging through an outlet pipe or cascading down a spillway, and wind. All generate random structural responses which are measured in ambient vibration tests. Subsequent Fourier analysis of the acquired data determines

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the frequency response characteristics of the dam, assuming uniform force over frequency, from which modal data are extracted. Mechanical exciters are employed for forced vibration tests. Rotating eccentric mass exciters generating sinusoidal forces are used more commonly, and, occasionally, random vibration generators. For sinusoidal excitation, the amplitude and phase of the harmonic responses of the dam are measured and compared with the force, over a range of frequencies. Transfer function spectra, the force–response relationships for the dam, are constructed, from which modal parameters are determined.

The control over the magnitude and location of the force in forced vibration tests ensures measurable responses above the ambient noise and enables control over the modes of vibration to be excited. The success of ambient vibration tests depends strongly on the magnitude and distribution of the ambient forces, neither of which can be controlled. Consequently, neither measurable responses nor the number of modes excited can be guaranteed.

The cost of forced vibration testing can be prohibitive, resulting in dam owners rejecting it as a beneficial part of a seismic safety evaluation. It is significantly more expensive than ambient vibration testing due to the use of the mechanical exciters. Two personnel are sufficient for ambient vibration tests, but for the handling, setting up and safe operation of the exciters, generally, two more are required, increasing personnel costs by about 70 per cent. The hire and transport of the exciters can be approximately 50 per cent of the total cost of the tests. For a two-week test period, with its associated post-test data processing and reporting, the cost of forced vibration tests can be double that of ambient vibration tests. Thus, ambient vibration testing becomes an attractive option despite its perceived disadvantages.

The suitability of ambient vibration tests depends on the objectives of the dynamic tests. Experimental data may be required to validate a new analytical technique. For example, hydrodynamic pressures were measured at Baitings Dam<sup>9</sup> for validating a Lagrangian fluid finite element.<sup>10</sup> A controlled force was essential for measurable pressures distinguishable from background noise, and force–response data were required for validating the numerical model. Generally, the high level of experimental control required for research is not necessary for engineering projects. Engineering analysis tends to employ accepted methodologies, so, usually, validation of the modal properties of a model of the dam–reservoir–foundation system is adequate.

This paper aims to demonstrate, using the case study of Claerwen Dam, that, with improved data processing, ambient vibration tests can offer a viable alternative to forced vibration tests.

## 2. DESCRIPTION OF CLAERWEN DAM

Claerwen Dam (Figure 1) is a mass concrete gravity dam constructed in the 1950s<sup>11</sup> in Mid-Wales, in the UK. 56 metres high and 325 metres long, it is founded on sound mudstone, a relatively stiff rock. The dam comprises twenty-three monoliths joined by shear keys, with the central fifteen acting as a spillweir. In plan, the dam is curved to a radius of about 607 metres with a convex upstream face. A series of concrete arches provides an access bridge across the crest of the dam. There are two inspection galleries; one 10.5 metres below crest level and another lower one following the ground level of the valley. Forced vibration tests could not be conducted on the dam as there were no suitable locations for mechanical exciters.

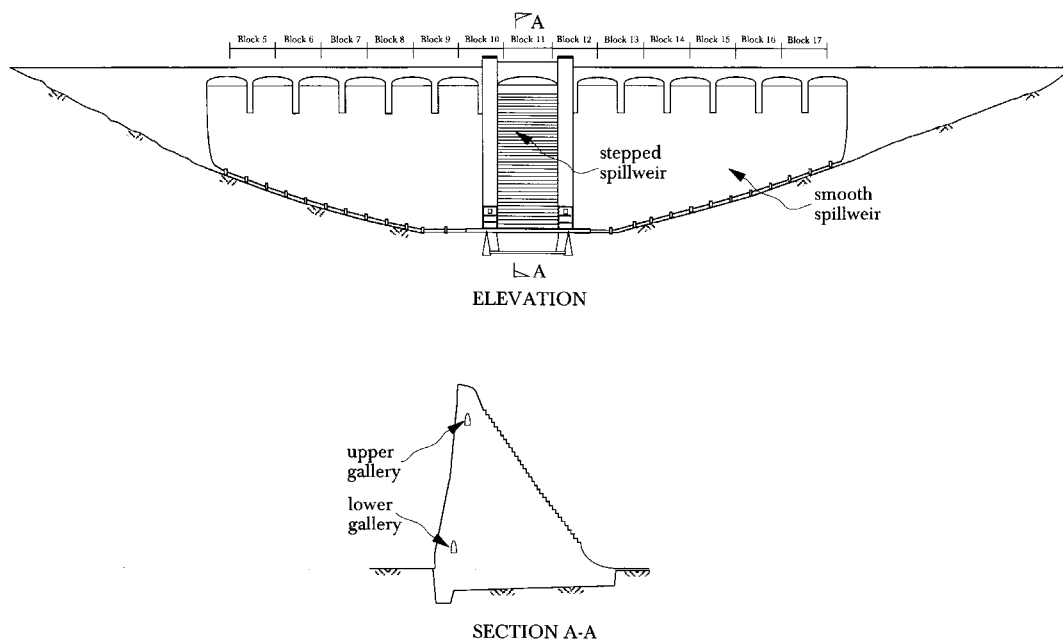


Figure 1. Claerwen dam — elevation and cross-section

### 3. AMBIENT VIBRATION TESTS

During the two-week test period, there were two main sources of ambient excitation. Initially, the reservoir was full (just above spillway level), and the excitation was caused by water cascading down the central stepped portion of the spillway. For the final three days, it was caused by water discharging through outlet-pipes at the base of the dam when the resulting change in the reservoir level was barely discernible. Intermittent light winds added to both sources of excitation. Peak accelerations of approximately  $22 \mu g$  were recorded for the 'outlet-pipe' vibrations, about three times greater than the 'spillway' vibrations. These vibrations were very low, resulting in significant noise in the measured signals.

Horizontal accelerations, radial to the curvature of the dam, were monitored in the upper gallery using four Sundstrand Q-Flex type QA-700 accelerometers. The reference accelerometer, was placed in Block 12 while the remaining "travelling" accelerometers were moved around other locations. Signals were conditioned via a custom designed unit with an amplifier having a maximum gain of 100, a 16th-order low-pass Butterworth filter with a cut-off frequency of 30 Hz, and a coil offset facility for measurement in any plane. The phase response of the filter is non-linear, although approximately linear up to a frequency of 20 Hz, then becoming more non-linear as the cut-off frequency is approached. The non-linear phase response is not an issue in the frequency analysis of ambient vibration data, as all signals from the travelling accelerometers are compared to those from the reference accelerometer, and all signals pass through similar filters cancelling the phase effects. Any small phase differences are measured by chain calibrating the travelling accelerometers relative to the reference accelerometer (all with their signal conditioning units) to acquire frequency response function (FRF) spectra for their relative magnitude and phase. These

differences are accounted for, later, during the frequency analysis of the data. Accelerometers were mounted on steel blocks ( $75 \times 55 \times 55$  cm.) which were supported on the concrete surfaces of the dam on three adjustable feet for approximate levelling. Any remaining tilt appeared as small DC offsets in the signals which were removed manually using the offset facility on the amplifier unit.

Response signals were digitised and stored on the hard disk of a portable computer, and data acquisition was controlled from a 12-bit Analogue to Digital Converter (ADC) board. Four channels of data were sampled using a simultaneous sample and hold (SSH) board offering the facility of further amplification up to a gain of 800. Small DC offsets in the signals not removed by visual examination were amplified through the SSH board. Signals were sampled at a rate of 128 Hz, to minimise aliasing errors, and amplified, generally, by a total gain of 10,000. Appropriate gains were determined by monitoring the signals before each data acquisition period, and were set to utilise 40 per cent of the input range of the ADC board to allow for fluctuations in the ambient forces and, therefore, the measured responses. High resolution power spectra with low variance and, thus, long data records were a prerequisite of the tests. Considering practical and time constraints, an optimum record length of three hours was selected for each measurement location. For each set of measurements, typically, an additional hour was required to lay out accelerometers and cables and to determine data acquisition settings.

During data acquisition, significant electronic drift was observed in the signals at the beginning of each day as the signal conditioning units warmed up after being switched on, but it disappeared once the temperature of the units had stabilised. A constant temperature could not be maintained by running the equipment overnight, as the power was supplied by a petrol generator which, for safety and security reasons, could not be left unattended. The electronic drift was relatively large compared to the low level response measurements and tended to cause the signals to exceed the range of the ADC board. Signals were monitored, and when the offset of the drift became excessive it was reset manually to zero.

#### 4. ANALYSIS OF TEST DATA

Fourier analysis requires random stationary data<sup>12</sup> in sufficient quantity to minimise the error in spectral estimates. Short-period records and non-stationary data result in poor quality spectra that limit the number of modes of vibration which can be identified. This section of the paper describes the improved methodology employed for the data processing for Claerwen Dam which progressed through four stages; the inspection and pre-processing of the raw data, stationarity testing, frequency analysis and modal analysis. For the first three stages, a computer program, ADPS, was written specifically for the processing and analysis of the long data records yielded from ambient vibration tests.

##### *4.1. Data inspection and pre-processing*

Data records totalling over 200 hours were acquired from the tests on Claerwen Dam. Visual inspection of all records for out-of-range data, instrumentation drift and DC offsets would have been impracticable. Using ADPS, this constraint was overcome by dividing each record into blocks of user-specified length for which the mean values (in Volts) were calculated and plotted. Blocks with data exceeding the input range of the ADC board ( $\pm 5$  Volts) were allocated a mean value of  $+5$ . A plot of the mean values for each record illustrated the tendency of the data, and

extreme values of  $+5$  highlighted any out-of-range data blocks. Hence, in this simple manner, records could be identified for trend and mean removal for subsequent analysis.

DC offsets were removed from data blocks by calculating and subtracting their mean value. Instrumentation drift was assumed, initially, to be linear over each data block. Linear trends were evaluated by fitting a straight line using the least squares fitting technique<sup>12</sup> and then subtracted. If subsequent Fourier analysis indicated significant low frequency components below the fundamental frequency of the dam, data were reprocessed to remove the low frequency trends using a high-pass Butterworth filter of user-specified order and cut-off frequency.

#### 4.2. Stationarity tests

Random data are defined as being stationary when their statistical properties over a series of short time intervals do not vary significantly from one interval to the next. The Claerwen data records were subjected to two stationarity tests. Each data record was tested to establish a stationary proportion for division into an ensemble of data blocks, for averaging in the frequency analysis. In addition, each data block was tested for stationarity also. Failed blocks were rejected from the overall stationarity test and the frequency analysis, thus, maximising the quality of the input data.

The run test<sup>12</sup> was employed for the stationarity tests. Each data record (and data block) was divided into  $n_d$  equal time intervals. The mean square value,  $x_i$ , for each interval was computed and aligned in sequence. The mean,  $\bar{x}$ , of the mean square values was computed. Each observation was classified as either  $+$  for  $x_i \geq \bar{x}$ , or  $-$  for  $x_i < \bar{x}$ . The sequence of plus and minus observations was examined to determine the number of runs,  $R$ , a run being defined as a sequence of identical observations that is preceded and followed by a different observation or no observation at all.

Next, the hypothesis that the sequence of observations was an independent set from a stationary random process was tested. A significance level,  $\alpha$ , for the hypothesis test was selected, and lower and upper limits,  $r_{1-\alpha/2}$  and  $r_{\alpha/2}$ , were calculated for the number of runs using the cumulative probability density function for the run distribution<sup>13</sup> which is given as

$$\Pr[r \leq r_z] = \frac{1}{\binom{m+n}{n}} \sum_{j=2}^{r_z} f_j, \quad (1)$$

where  $m \leq n$  and there are  $m$  pluses and  $n$  minuses or vice versa,

$$f_j = 2 \binom{m-1}{k-1} \binom{n-1}{k-1} \quad \text{for even } j, \text{ with } j = 2k, \quad (2)$$

and

$$f_j = \binom{m-1}{k-1} \binom{n-1}{k-2} + \binom{m-1}{k-2} \binom{n-1}{k-1} \quad \text{for odd } j, \text{ with } j = 2k-1. \quad (3)$$

For the hypothesis to be accepted  $r_{1-\alpha/2} < R \leq r_{\alpha/2}$ .

The result of the test depends on the length and number of the time intervals. The time interval should encompass a number of cycles of response of the fundamental mode of vibration to ensure

that the mean square value for that mode is fairly consistent between intervals. Too short an interval will result in a failed stationarity test with the fundamental modal response appearing as a trend. However, dividing a record into a smaller number of longer time intervals increases the chances of passing the test as variations in consecutive mean square values are smoothed.

#### 4.3. Frequency and modal analysis

Autopower spectra were estimated for the responses from all accelerometer locations. Cross-power, FRF and coherence spectra were estimated for the responses from all travelling accelerometer locations relative to the reference location. Data records were divided into smaller blocks for spectral averaging. Hanning windows were applied to the data blocks before Fourier transformation into the frequency domain, effectively reducing the resolving power of the analysis. However, 90 per cent of the stability of the power lost was regained by overlapping data blocks by 50 per cent,<sup>15</sup> thus doubling the number of FFT operations.

Natural frequencies of the dam were identified from resonant peaks in the autopower spectra, and modal damping factors were estimated by applying the half-power bandwidth method<sup>12</sup> to the peaks. As autopower spectra from ambient vibration tests tend to be contaminated by noise and variance hindering the recognition of modal peaks, crosspower and coherence spectra were used additionally to support the identification process. Where a mode of vibration contributes to the response at both the reference and travelling accelerometer locations, a better defined modal peak becomes evident in the crosspower spectrum. The coherence function provides a measure of the repeatability of the spectral estimates, for autopower and crosspower, and it is expressed<sup>12</sup> as:

$$\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f) \cdot G_{yy}(f)}, \quad (4)$$

where  $\gamma_{xy}^2(f)$  is the coherence function between  $x(t)$  and  $y(t)$ ;  $x(t)$ ,  $y(t)$  are the two response records;  $G_{xy}(f)$  is the cross-power function between  $x(t)$  and  $y(t)$ ;  $G_{xx}(f)$  is the autopower function for  $x(t)$ ; and  $G_{yy}(f)$  is the autopower function for  $y(t)$ .

Its value varies between zero and unity. In traditional modal testing, for example, in the aerospace industry, values close to unity are sought to demonstrate high quality measurements. However, such high values often may not be associated with ambient responses from large dams, so the coherence function is employed as a relative measure of the quality of the data. Peaks in the coherence spectra coinciding with modal peaks in the autopower and crosspower spectra serve as further evidence of a mode of vibration.

The FRFs compare the responses from the travelling accelerometers to the reference accelerometer, giving their relative magnitude and phase difference. For each mode of vibration, spectral values for the magnitude and phase at the natural frequency were extracted from the FRF spectra for all measurement locations, and, then, grouped together to generate the mode shape. At this stage of the analysis the modal parameters were corrected for differences in the accelerometer calibrations. For normal modes, points on the mode shapes can then be recognised as being either in-phase or 180° out-of-phase with each other. Strictly, this methodology should be applied only to spectra showing well separated modes, as the modal parameters from spectra with closely coupled modes can be distorted due to overlapping modal responses.

To maximise the accuracy of the natural frequencies, mode shapes and damping factors, selected parameters for the frequency analysis required optimisation. The number of spectral

averages and the spectral resolution are important parameters, and, for a finite data record, they are directly related, but they have an conflicting influence on the error in the spectral estimates. For the frequency analysis of the Claerwen data, these parameters were optimised to minimise the errors in the autopower spectra, and, thus, improve indirectly the accuracy of the FRF spectral estimates.

The normalised root-mean-square (rms) error,  $\varepsilon$ , for the autopower spectral estimates were evaluated from the following expression<sup>12</sup>:

$$\varepsilon^2[\hat{G}_{xx}(f)] = \varepsilon_r^2[\hat{G}_{xx}(f)] + \varepsilon_b^2[\hat{G}_{xx}(f)] \quad (5)$$

The first term, a variance term, is the square of the normalised random error,  $\varepsilon_r$ , describing the random portion of the error, and the second term, a bias term, is the square of the normalised bias error,  $\varepsilon_b$ , describing the systematic portion of the error. Equation (5) is expressed more fully as

$$\varepsilon^2[\hat{G}_{xx}(f)] = \frac{1}{B_e T_r} + \left( \frac{B_e^2 G''_{xx}(f)}{24 G_{xx}(f)} \right)^2, \quad (6)$$

where  $G_{xx}(f)$  is the true autopower spectral value at  $f$ ;  $G''_{xx}(f)$  is the true second derivative of  $G_{xx}(f)$ ;  $\hat{G}_{xx}(f)$  is the estimate for  $G_{xx}(f)$ ;  $T_r$  is the total record length (secs) which may be split into  $n_d$  smaller blocks of length  $T$ , i.e.,  $n_d T \approx n_d / B_e$ ; and  $B_e$  is the frequency resolution bandwidth (Hz) approximately equal to  $1/T$ .

Improved spectral estimates for autopower can be achieved by minimising the normalised rms error,  $\varepsilon$ . Equation (6) demonstrates that for given record length  $T_r$ , the random error is inversely proportional to the square root of the frequency resolution bandwidth,  $B_e$ , while the bias error is directly proportional to its square. Any attempt to reduce the random error by increasing  $B_e$  will have the effect of increasing the bias error. The record length,  $T_r$ , could be increased to reduce the random error without affecting the bias error. In practice, this is not possible, as finite data records would have been acquired and fully utilised already. To determine the optimum frequency resolution for a given record length, preliminary analyses are required to establish a balance between the two errors.

A small resolution bandwidth aids the identification of closely coupled modes, but, for a finite record, it can result in high variance which will appear as noise in the spectrum tending to obscure individual peaks. On the other hand, increasing the resolution bandwidth will smooth the spectrum, but the bias error will increase and any closely coupled modes of vibration may merge into one peak.

Normalised random errors can be calculated easily, but bias errors require knowledge of the true spectral values and their second derivatives, which obviously are not available. An approximation<sup>12</sup> for the bias error at a resonant peak, for the assumption of white noise input over its frequency range, can be expressed as:

$$\varepsilon_b[\hat{G}_{xx}(f_r)] = -\frac{1}{3} \left( \frac{B_e}{B_r} \right)^2, \quad (7)$$

where  $f_r$  is the resonant frequency,  $B_r$  the half-power point bandwidth of the resonant peak.

The bias error at a resonant peak will always be negative underestimating the peak spectral value and, therefore, overestimating the modal damping factor from the half-power

bandwidth method. However, bias error is negligible for sufficiently small frequency resolution bandwidths.

## 5. TEST RESULTS

### 5.1. *Results of stationarity tests*

The dam was excited, initially, by spillweir forces and, then, by outlet-pipe forces. Both forces were fairly consistent during the acquisition of each record, as the flow of water over the spillweir remained almost constant, and the change in the head of water above the outlet-pipes was negligible. Therefore, stationary ambient responses were expected.

Preliminary analyses determined the optimum number of samples per data block as 4096. For the spillweir responses, overall stationarity was proved to 1 per cent significance for a maximum of 256 consecutive blocks (approximately two hours), but longer records failed the stationarity tests. Sometimes, stationarity could be proved for only two of the four channels of data acquired simultaneously. However, the remaining channels were used in the frequency analysis, as they had only just failed the hypothesis of stationarity. The tests on the individual data blocks employed 32 time intervals of one second, each containing 128 samples and encompassing approximately six cycles of the fundamental modal response. Generally, no more than 10 per cent of data blocks failed the stationarity tests. Tests on the outlet-pipe vibration data were more successful, proving overall stationarity for longer periods and all channels of data acquired simultaneously.

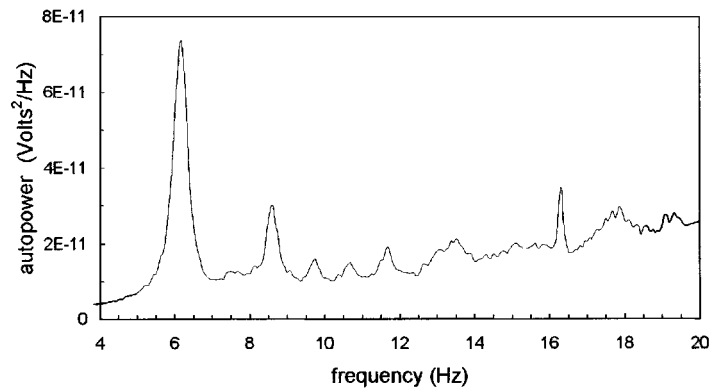
It is believed that stationary responses were achieved because the wind excitation had been insignificant. If, stronger and more variable winds had occurred, stationarity for such long periods would have been difficult to prove. Therefore, it is recommended that measurements of wind speed and direction are acquired simultaneously with the response measurements, so that data acquired on different occasions, but under similar conditions, can be grouped together into stationary sets for frequency analysis. By leaving equipment on site, long term, and by remote monitoring, data can be acquired under advantageous conditions over a longer period, generating sufficient stationary data.

### 5.2. *Results of frequency analysis*

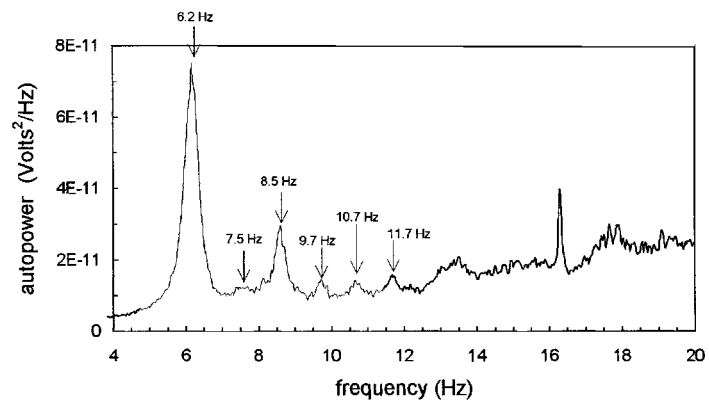
Figure 2 shows three autopower spectra from the same data record for the reference location under spillweir excitation. Table I gives their frequency resolution, number of samples per data block, number of blocks per record and normalised random and bias errors for the fundamental mode. The bias error is negligible for all spectra, but the increase in variance with decreasing resolution is clear. It appears that a 0.0625 Hz frequency resolution and a 2048 block length yield the optimum spectrum. However, the overall stationarity tests failed for this block length. By doubling the block length, stationarity was proved, so a 0.0312 Hz frequency resolution and a 4096 block length were used in the main frequency analysis. Although the autopower spectra indicate the presence of significant noise, six resonant peaks can be identified below 14 Hz.

A typical autopower spectrum for the reference location under outlet-pipe excitation (Figure 3) has better defined resonant peaks than its corresponding spillweir spectrum (Figure 2b) enabling the identification of another resonance below 14 Hz. The noise level clearly is lower. The small differences in the natural frequencies from both excitation sources are not worthy of comment.

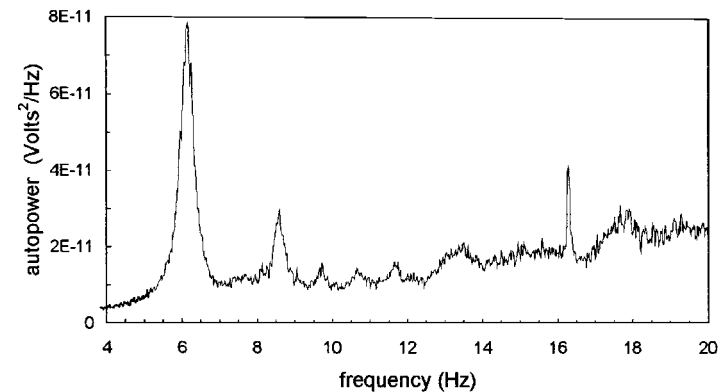




(a) 0.0625 Hz Resolution



(b) 0.0312 Hz Resolution



(c) 0.0156 Hz Resolution

Figure 2. Spillweir excitation — autopower spectra at the reference location

Table I. Errors and frequency resolution for spillweir autopower spectra

Figure no.	Frequency resolution (Hz)	No. of samples per block	No of blocks $n_d$	Normalised random error (%)	Bias error at $f_1$ (%)
2(a)	0.0625	2048	512	4.4	0.5–1.4
2(b)	0.0312	4096	256	6.3	0.1–0.4
2(c)	0.0156	8192	128	8.8	0.03–0.09

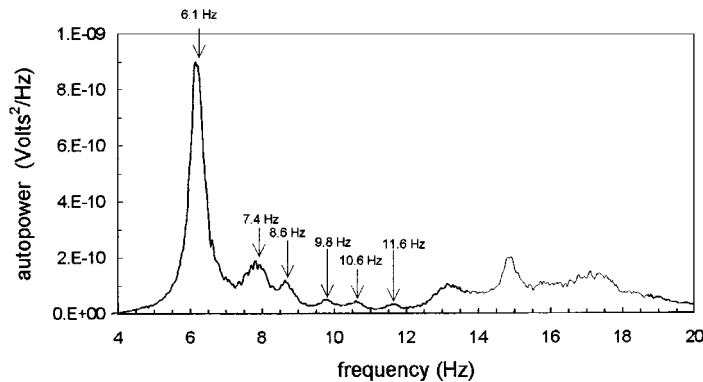


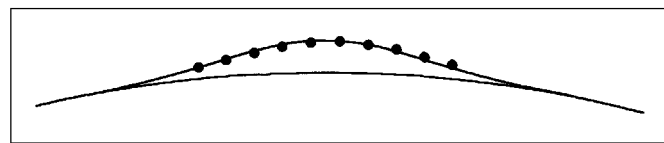
Figure 3. Outlet-pipe excitation — Autopower spectrum at the reference location 0.0312 Hz resolution

The greater variance in the spillweir response spectrum reflect the results of the stationarity tests indicating lower quality data compared to the outlet-pipe response data. Generally, resonances in the autopower spectra are well defined, for both excitation forces, as a result of the low variance achieved by using a sufficient quantity of stationary data. The quality of these power spectra shows a significant improvement on results published previously.<sup>1–4</sup>

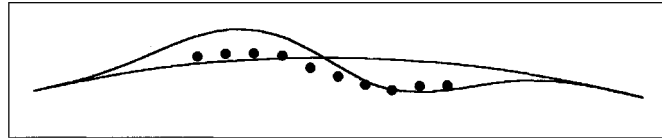
### 5.3. Results of modal analysis

Natural frequencies, mode shapes, and modal damping factors for six lateral modes of vibration are presented in Figure 4 and Table II. Although measurements were taken along the central portion of the dam only, and some relatively low coherence was measured, the mode of vibration associated with each frequency can be identified clearly. Some mode shapes appear distorted, probably as a result of the overlapping of modal responses and the low measured coherence. A typical coherence spectrum is shown in Figure 5. Good coherence was measured for the fundamental mode. Peaks in the coherence are relatively stronger for the symmetric modes compared to the anti-symmetric modes. This is not a surprising result as the sources of excitation were concentrated at the centre of the dam, exciting the symmetric modes more strongly.

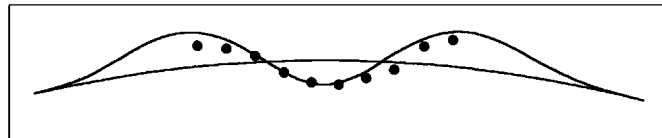
The estimated modal damping factors are equivalent values for the combined energy dissipation mechanisms present in the dam–reservoir–foundation system. The contribution from



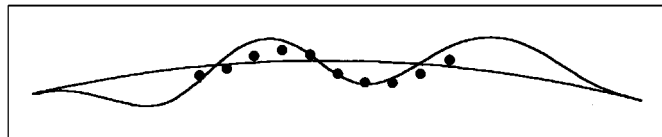
Mode 1: Frequencies - Measured 6.1-6.2 Hz - FE Model f45d21 5.9 Hz



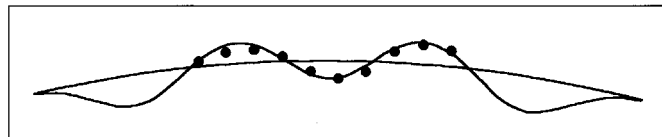
Mode 2: Frequencies - Measured 7.4-7.7 Hz - FE Model f45d21 7.5 Hz



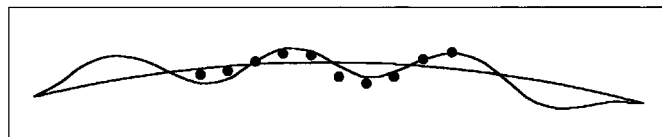
Mode 3: Frequencies - Measured 8.5-8.6 Hz - FE Model f45d21 9.0 Hz



Mode 4: Frequencies - Measured 9.7-9.8 Hz - FE Model f45d21 10.5 Hz



Mode 5: Frequencies - Measured 10.6-10.7 Hz - FE Model f45d21 12.0 Hz



Mode 6: Frequencies - Measured 11.6-11.8 Hz - FE Model f45d21 13.6 Hz

Figure 4. Measured and theoretical mode shapes and natural frequencies (● measured mode shape from tests, ——— computed mode shape from F.E. model)

foundation damping would be expected to be small as the foundation rock is relatively stiff resulting in insignificant dam–foundation interaction. However, a notable contribution from reservoir radiation damping could be expected due to the effects of water compressibility.<sup>14</sup> Compressibility effects are believed to become apparent when the ratio of the fundamental frequency of the reservoir,  $f_w$ , to the fundamental frequency of the dry dam,  $f_d$ , is less than 1.5. For

Table II. Measured frequencies of vibration and damping factors

Mode	Frequency (Hz)	Damping (spillweir) %	Damping (outlet pipe) %
1st symmetric	6.1–6.2	2.4–4.6	4.4
1st anti-symmetric	7.4–7.7	5.2	5.9
2nd symmetric	8.5–8.6	2.5–3.9	—
2nd anti-symmetric	9.7–9.8	—	—
3rd symmetric	10.6–10.7	—	—
3rd anti-symmetric	11.6–11.8	—	—

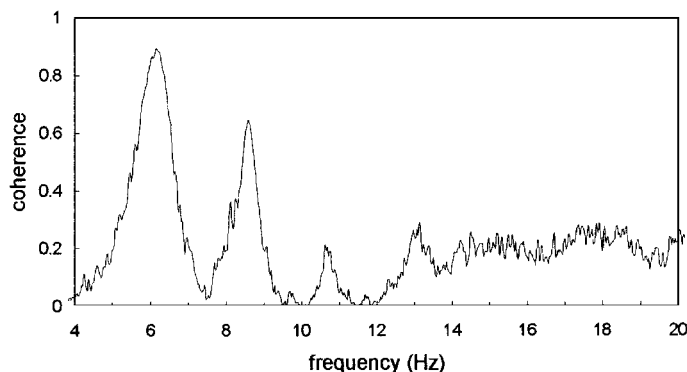


Figure 5. Spillweir excitation — Typical coherence spectrum, upper gallery 0.0312 Hz resolution

a wide valley, like the one at Claerwen Dam,  $f_w$ , is given by

$$f_w = \frac{c}{4H}, \quad (8)$$

where  $c$  is the speed of sound in water (1450 m/s) and  $H$  is the depth of reservoir in metres.

$f_d$  for Claerwen Dam was estimated as 7.3 Hz, and  $f_w$  as 6.5 Hz, giving a ratio of 0.9 for  $f_w/f_d$ . Low damping was measured, not reflecting the presence of any significant reservoir radiation damping. This result is consistent with previous tests for which the presence of such little damping was noted as interesting, given the large potential for radiation damping through the foundation rock and reservoir.<sup>8</sup> Furthermore, the damping factors, measured for Claerwen Dam, are overestimates due to the presence of noise, the application of the half-power bandwidth method to overlapping resonant peaks, bias errors and windowing. The latter two effects are minimised by the high spectral resolution employed. Errors in damping estimates from ambient vibration data are discussed in detail in Reference 16.

## 6. FINITE ELEMENT MODELLING AND ANALYSIS

EACD-3D,<sup>17</sup> a computer program developed for the three-dimensional seismic analysis of concrete dams was used for the finite element analysis. The dam and foundation are idealised with

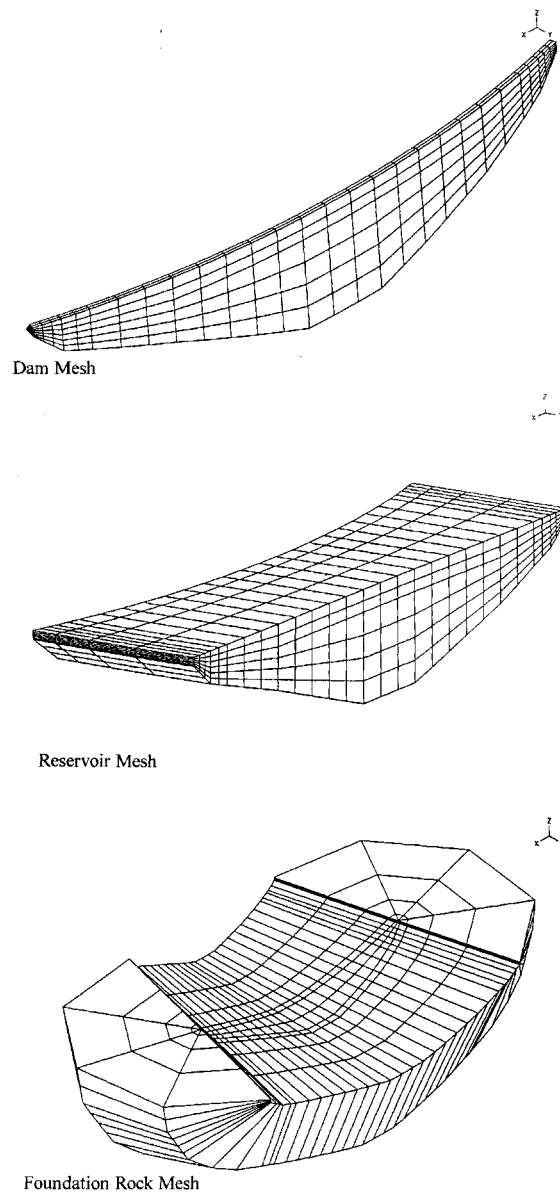


Figure 6. Finite element meshes for Claerwen dam

solid elements, and the reservoir with compressible fluid elements adjacent to the dam and an adjoining infinite uniform channel. The dam is characterised by its modulus of elasticity, Poisson's ratio, density and damping properties, and the flexibility of the foundation is characterised, but not its damping and inertial properties. A wave reflection coefficient characterises the absorptive properties of the reservoir boundaries. The analytical procedure is based on the substructure method of analysis and frequency domain analysis.

Table III. Identifiers of F.E. models and elastic moduli

Name	Foundation rock modulus (GPa)	Dam modulus (GPa)
f22d30	22	30
f30d30	30	30
f30d25	30	25
f30d21	30	21
f35d21	35	21
f45d21	45	21

Table IV. Natural frequencies of vibration from F.E. models

Mode	f22d30	f30d30	f30d25	f30d21	f35d21	f45d21
1st symmetric	5.9	6.2	6.0	5.7	5.8	5.9
1st anti-symmetric	7.5	8.1	7.6	7.2	7.3	7.5
2nd symmetric	9.0	9.8	9.2	8.7	8.8	9.0
2nd anti-symmetric(1)	10.9	11.6	10.8	10.2	10.3	10.5
2nd anti-symmetric(2)	12.6	11.5	10.8	10.2	10.3	10.5
3rd symmetric	12.7	13.4	12.5	11.7	11.8	12.0
3rd anti-symmetric	15.5	15.2	14.2	13.3	13.3	13.6

The finite element meshes for the dam, reservoir and foundation are shown in Figure 6. Construction records<sup>11</sup> for Claerwen Dam indicate a characteristic 28-day compressive strength for the concrete of about 22 MPa, for which the current elastic modulus was estimated as 21–35 GPa.<sup>18</sup> The foundation rock, sound mudstone, has an elastic modulus ranging typically from 20–50 GPa.<sup>19</sup> The following material properties were employed for the model:

Dam concrete	Density	= 2380 kg/m <sup>3</sup>
	Modulus of elasticity	= 21, 25 and 30 GPa
	Poisson's ratio	= 0.2
	Constant hysteretic damping factor	= 0.1 (equivalent to 5% viscous damping)
Foundation rock	Modulus of elasticity	= 22, 30, 35 and 45 GPa
	Poisson's ratio	= 0.25
Reservoir	Speed of sound in water	= 1450 m/s
	Wave reflection coefficient	= 1.0 (i.e. no absorption)

Six combinations of concrete and foundation moduli were employed. Each model is identified in Table III.

Modal and response analyses were run for each model. EACD-3D computes mode shapes for the dam–foundation system only. They are employed as generalised co-ordinates for which frequency response functions are formulated including the reservoir effects. Natural frequencies of the dam–foundation–reservoir system (Table IV) were determined from resonant peaks in plots of the frequency response functions. The natural frequencies of the model were not very sensitive to

changes in the concrete and foundation moduli, especially for the lower modes. The lateral mode shapes for the dam–foundation system were compared with those measured from the tests for the full reservoir (Figure 4). Two modes with very close frequencies were computed, both showing the second anti-symmetric lateral mode shape. One was the pure second anti-symmetric lateral mode, and the other a combination of that and a cross-stream sway mode caused by the curvature of the dam.

Response analyses of the different models were run to generate theoretical autopower spectra for comparison with the measured outlet-pipe spectra for the reference location on the dam. The purpose of these analyses was to compare the characteristics of the experimental and theoretical responses and the spacing of the modes of vibration. The ground motion input was an acceleration time history with white-noise characteristics i.e. having energy distributed uniformly, with an amplitude of  $1.0 \text{ m/s}^2$ , over a frequency range 0–32 Hz, and its duration was continued until smooth autopower spectra were evaluated from the response time histories. The ground motion was applied in the stream direction only and was intended to be representative of the outlet-pipe excitation, but by no means an accurate representation. Ten modes of vibration were included in the analysis.

Displacement time histories for the centre of the dam crest were examined for all models. The largest peak response occurred for model f22d30, 32 per cent greater than the smallest peak response in model f30d30. The results of the response analysis demonstrated that, although the computed natural frequencies are fairly insensitive to changes in the concrete and foundation moduli, the response amplitudes can vary significantly. These differences highlight the importance of parametric studies on finite element models where uncertainties exist in the input parameters employed. Such studies can be very useful in assessing the reliability of the results from the seismic analysis of a model.

## 7. COMPARISON OF MEASURED AND THEORETICAL MODES

Natural frequencies from the finite element model f45d21 (Table IV) matched the measured values most closely (Table II), showing good agreement with differences of up to 8 per cent for the lowest four modes and up to 17 per cent for the higher modes. The resolution of the finite element mesh is not sufficiently refined for the higher modes overestimating their stiffness and, thus, predicting higher frequencies.

Theoretical and experimental mode shapes for the six lowest lateral modes of vibration are presented in Figure 4. Only one mode shape for the second anti-symmetric mode was measured from the field tests, but it is distorted relative to the other measured mode shapes, which could be a result of the presence of the two similar modes responding out of phase with each other. Although good correlation could not be achieved for all modes, generally, it is reasonable, with experimental and theoretical nodes and anti-nodes occurring at approximately the same points.

Measured and theoretical autopower spectra are compared in Figure 7. They have been scaled to give the same spectral value at the fundamental frequency. The characteristics of the spectrum from model f45d21 matches the measured spectrum most closely, with the lowest three resonant peaks spanning a similar frequency range and with the second resonance being more apparent for this model. Given that the base acceleration input for the response analysis was not an exact representation of the outlet-pipe excitation, the characteristics of the theoretical and experimental spectra are remarkably similar. Both show a dominant response in the first three modes dropping

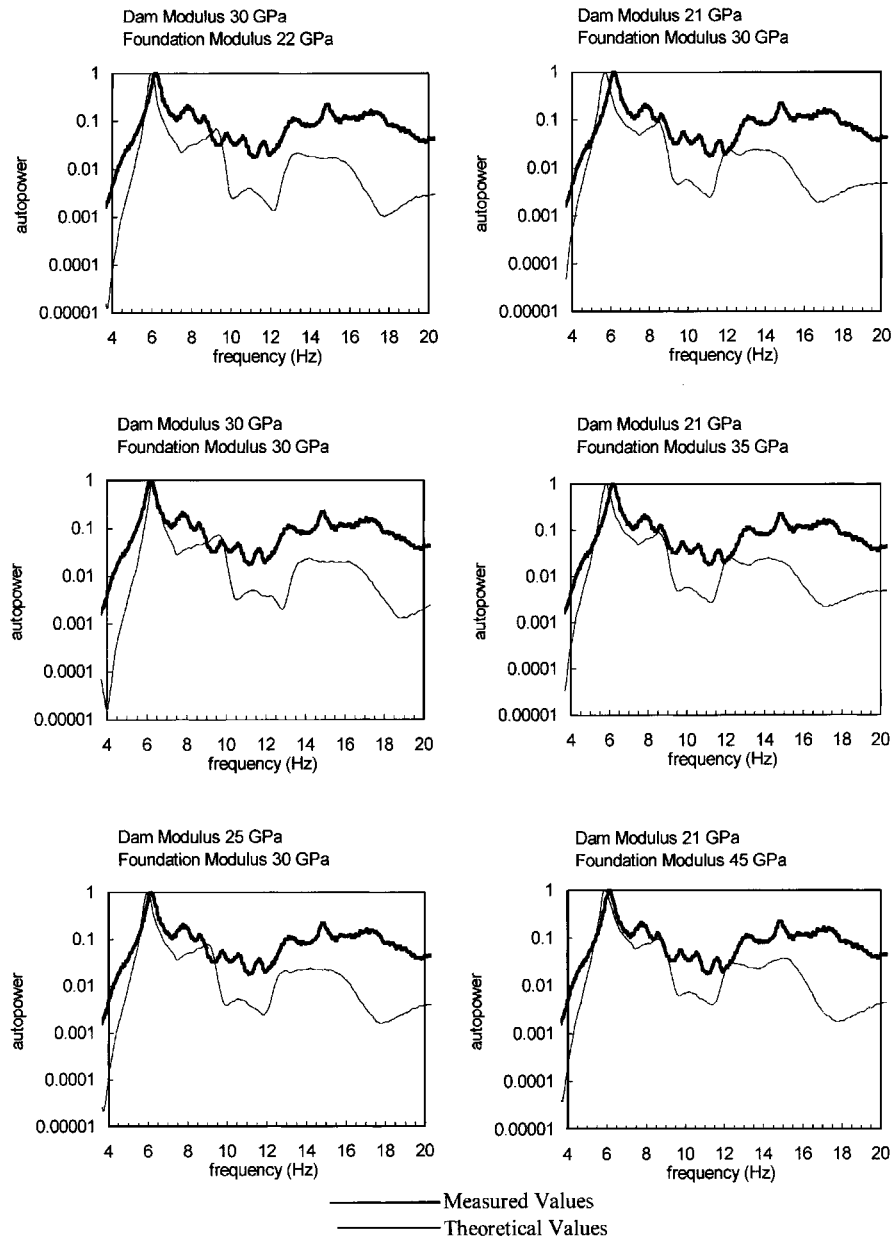


Figure 7. Measured 'outlet-pipe' and theoretical autopower spectra at reference location

off to a significantly lower response for subsequent modes. However, the theoretical modal responses above 12 Hz do not build up again as much as the experimental ones.

The good correlation between natural frequencies and mode shapes for the theoretical and measured results for the lowest modes of vibration validate the finite element model of Claerwen



Dam. As these lower modes would dominate the earthquake response of the dam, the finite element model should be adequate for its seismic analysis as part of a seismic risk assessment. The higher modes could be modelled more accurately with a finer mesh resolution, but it is arguable whether further refinement is necessary for an engineering analysis given the other uncertainties in the numerical modelling. Doubt exists over, for example, the current modulus of elasticity for the concrete, the validity of a global modulus of elasticity for variable foundation conditions, the assumption of uniform seismic input along the dam, the modelling of wave absorption at reservoir boundaries, and the modelling for foundation radiation damping. The modelling of all of these factors depends on engineering judgement and can influence significantly the computed response for the dam. The level of refinement of the model must be balanced against the degree of confidence in the input parameters for the analysis. The basic modal characteristics of the numerical model can be validated through dynamic testing, but the sensitivity of the response of the model to changes in these parameters can only be investigated through parametric studies.

## 8. CONCLUSIONS

- This study demonstrates that ambient vibration testing can be effective in validating numerical models of dams, providing that test data are acquired and processed carefully to improve the quality of the data for Fourier analysis. Employing the procedures described in this paper, ambient vibration testing can provide a viable alternative to forced vibration testing.
- Rejection of non-stationary data improves the quality of data for Fourier analysis, increasing the accuracy of spectral estimates for frequency response functions and, thus, the accuracy of mode shapes.
- Long data records should be acquired to increase the number of spectra available for averaging, thereby minimising errors in frequency response spectrum estimates.
- Stationary random vibration data are required for the Fourier analysis. Outlet-pipe and spillweir excitation can yield stationary data. However, for variable wind conditions, the simultaneous acquisition of wind measurements is recommended, so that data acquired under different conditions can be grouped into stationary sets for Fourier analysis.
- Mode shapes and natural frequencies computed from EACD-3D corresponded well with those measured from the ambient vibration tests.

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